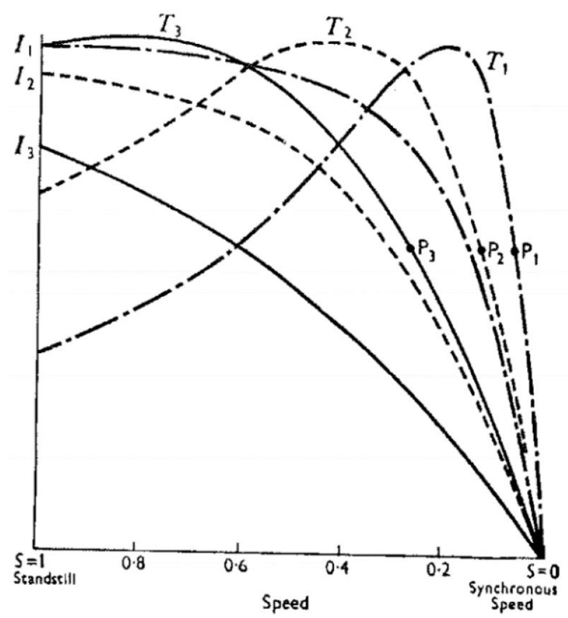
EE3124 Tutorial 6 (Solution)
Induction Machine

Name: **Student No.:**

Q1 - Sketch the torque-speed and current-speed characteristics of a three-phase wound-rotor induction motor when using external resistance control. Hence, explain why both high starting torque and low starting current can be achieved simultaneously.

Solution



The starting torque depends on the applied voltage, starting current and the corresponding power factor. When external resistance is properly added during the starting, the improvement in power factor is dominant to the reduction of starting current. Hence, both reduction of starting current and improvement of staring torque can be achieved simultaneously.

Q2 - A three-phase, 380 V, 50 Hz, 8-pole star-connected wound rotor induction motor has a stator impedance of (0.1+j 0.55) Ω per phase, and a rotor impedance of (0.2+j 0.55) Ω per phase referred to the stator side. Ignoring the magnetizing current ($I_{XM} = 0$), determine the required external resistance (referred to the stator side) which should be inserted in each rotor phase to achieve the maximum starting torque. Hence, calculate this maximum starting torque.

Given:

The air gap power $P_{AG} = \dfrac{3V_{TH}^2 R_2/s}{\left(R_{TH} + R_2/s\right)^2 + \left(X_{TH} + X_2\right)^2}$ The induced torque $\tau_{ind} = \dfrac{P_{AG}}{\omega_{sync}}$

The maximum power transfer occurs when $\dfrac{R_2}{s} = \sqrt{R_{TH}^2 + \left(X_{TH} + X_2\right)^2}$

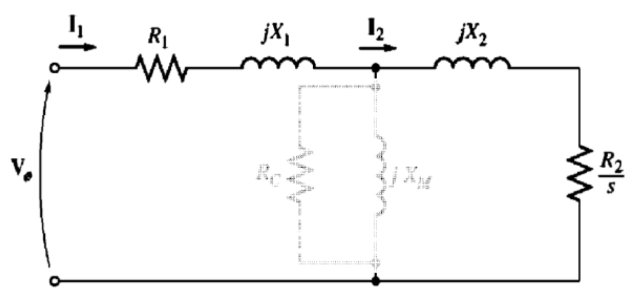
Solution

$$N_s = \frac{rpm}{60} = \frac{\frac{50 \times 60}{4}}{60} = \frac{750}{60} = 12.5\, rps$$

In the per phase equivalent circuit, $V_\phi = \dfrac{V_{LL}}{\sqrt{3}} = \dfrac{380}{\sqrt{3}} = 219.4\text{ V V}$

When the motor starts, slip s=1, $R_2^{'} = \sqrt{R_1^2 + \left(X_1 + X_2\right)^2}$ and $R_2^{'} = R_2 + R_e$

where R_e is the external rotor resistance.



$$R_2 + R_e = \sqrt{0.1^2 + (0.55 + 0.55)^2}$$

$$0.2 + R_e = 1.1045$$

$$R_e = 0.9045$$

$$\tau_{ind} = \frac{P_{AG}}{\omega_{sync}} = \frac{3V_{TH}^2 R_2/s}{(2\pi N_s)\left[\left(R_{TH} + R_2/s\right)^2 + \left(X_{TH} + X_2\right)^2\right]}$$

$$= \frac{3 \times (219.4)^2 \times 1.1045}{(2\pi N_s) \times \left[(0.1 + 1.1045)^2 + (0.55 + 0.55)^2\right]}$$

$$= \frac{159499.8289}{(78.5398) \times \left[(1.4508) + (1.21)\right]}$$

$$= 763.2348\ \text{N} \cdot \text{m}$$

Q3 - A 220-V three-phase six-pole 50-Hz induction motor is running at a slip of 3.5 percent. Find:

(a) The speed of the magnetic fields in revolutions per minute.

(b) The speed of the rotor in revolutions per minute.

(c) The slip speed of the rotor.

(d) The rotor frequency in hertz.

Solution

(a) The speed of the magnetic fields is

$$n_{\text{sync}} = \frac{120 f_{2e}}{P} = \frac{120(50 \text{ Hz})}{6} = 1000 \text{ r/min}$$

(b) The speed of the rotor is

$$n_m = (1 - s)\, n_{\text{sync}} = (1 - 0.035)(1000 \text{ r/min}) = 965 \text{ r/min}$$

(c) The slip speed of the rotor is

$$n_{\text{slip}} = s n_{\text{sync}} = (0.035)(1000 \text{ r/min}) = 35 \text{ r/min}$$

(d) The rotor frequency is

$$f_{re} = \frac{n_{\text{slip}} P}{120} = \frac{(35 \text{ r/min})(6)}{120} = 1.75 \text{ Hz}$$

Q4 - A two-pole, 50-Hz induction motor supplies 15 kW to a load at a speed of 2950 r/min.

(a) What is the motor's slip?

(b) What is the induced torque in the motor in N • m under these conditions?

(c) What will the operating speed of the motor be if its torque is doubled?

(d) How much power will be supplied by the motor when the torque is doubled?

Solution

***(a)* The synchronous speed of this motor is**

$$n_{\text{sync}} = \frac{120 f_{se}}{P} = \frac{120(50 \text{ Hz})}{2 \text{ poles}} = 3000 \text{ r/min}$$

Therefore, the motor's slip is

$$s = \frac{n_{\text{sync}} - n_{\text{m}}}{n_{\text{sync}}} (\times\ 100\%)$$

$$= \frac{3000\ \text{r/min} - 2950\ \text{r/min}}{3000\ \text{r/min}} (\times\ 100\%)$$

$$= 0.0167 \text{ or } 1.67\%$$

(b) The induced torque in the motor must be assumed equal to the load torque, and P_{conv} must be assumed equal to P_{load}, since no value was given for mechanical losses. The torque is thus

$$\tau_{\text{ind}} = \frac{P_{\text{conv}}}{\omega_{\text{m}}}$$

$$= \frac{15\ \text{kW}}{(2950\ \text{r/min})(2\pi\ \text{rad/r})(1\ \text{min}/60\ \text{s})}$$

$$= 48.6\ \text{N} \cdot \text{m}$$

(c) In the low-slip region, the torque–speed curve is linear, and the induced torque is directly proportional to slip. Therefore, if the torque doubles, then the new slip will be 3.33 percent. The operating speed of the motor is thus

$$n_m = (1 - s)n_{\text{sync}} = (1 - 0.0333)(3000\ \text{r/min}) = 2900\ \text{r/min}$$

(d) The power supplied by the motor is given by

$$P_{\text{conv}} = \tau_{\text{ind}}\omega_m$$

$$= (97.2\ \text{N} \cdot \text{m})(2900\ \text{r/min})(2\pi\ \text{rad/r})(1\ \text{min}/60\ \text{s})$$

$$= 29.5\ \text{kW}$$

Q5 – A 460-V 60-Hz four-pole Y-connected induction motor is rated at 25 hp. The equivalent circuit parameters are

$R_1 = 0.15$	$R_2 = 0.154$	$X_M = 20$
$X_1 = 0.852$	$X_2 = 1.066$	
$P_{F\&W} = 400$ W	$P_{misc} = 150$ W	$P_{core} = 400$ W

For a slip of 0.02, find

(a) The line current I_L

(b) The stator power factor

(c) The rotor power factor

(d) The rotor frequency

(e) The stator copper losses P_{SCL}

(f) The air-gap power P_{AG}

(g) The power converted from electrical to mechanical form P_{conv}

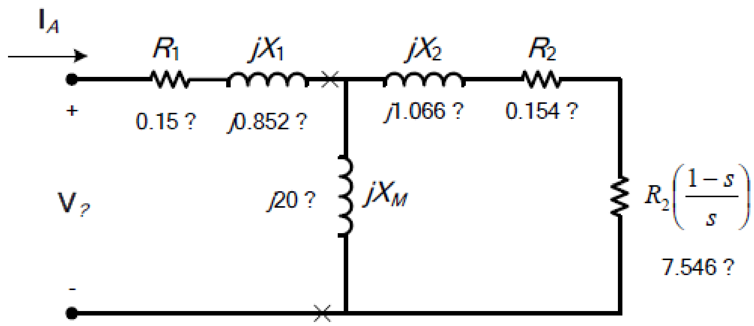
(h) The induced torque τ_{ind}

(i) The load torque τ_{load}

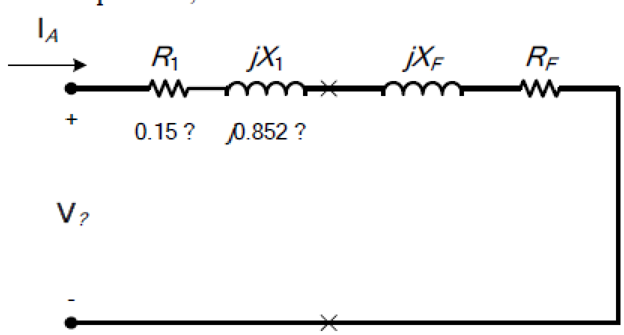
(j) The overall machine efficiency

(k) The motor speed in revolutions per minute and radians per second

Solution



(a) The easiest way to find the line current (or armature current) is to get the equivalent impedance Z_F of the rotor circuit in parallel with jX_M, and then calculate the current as the phase voltage divided by the sum of the series impedances, as shown below.



The equivalent impedance of the rotor circuit in parallel with jX_M is:

$$Z_F = \frac{1}{\frac{1}{jX_M} + \frac{1}{Z_2}} = \frac{1}{\frac{1}{j20\ \Omega} + \frac{1}{7.70 + j1.066}} = 6.123 + j3.25 = 6.932\angle 28.0^\circ\ \Omega$$

The phase voltage is $460/\sqrt{3} = 266$ V, so line current I_L is

$$I_L = I_A = \frac{V_\phi}{R_1 + jX_1 + R_F + jX_F} = \frac{266\angle 0^\circ\ \text{V}}{0.15\ \Omega + j0.852\ \Omega + 6.123\ \Omega + j3.25\ \Omega}$$

$$I_L = I_A = 35.5\angle -33.2^\circ\ \text{A}$$

(b) The stator power factor is

$$\text{PF} = \cos(33.2°) = 0.837 \text{ lagging}$$

(c) To find the rotor power factor, we must find the impedance angle of the rotor

$$\theta_R = \tan^{-1}\frac{X_2}{R_2/s} = \tan^{-1}\frac{1.066}{7.70} = 7.88°$$

(d) The rotor frequency is

$$f_r = sf_s = (0.02)(60 \text{ Hz}) = 1.2 \text{ Hz}$$

Therefore the rotor power factor is

$$\text{PF}_R = \cos 7.88° = 0.991 \text{ lagging}$$

(e) The stator copper losses are

$$P_{\text{SCL}} = 3I_A{}^2R_1 = 3(35.5 \text{ A})^2(0.15\ \Omega) = 567 \text{ W}$$

(f) The air gap power is $P_{\text{AG}} = 3I_2{}^2\frac{R_2}{s} = 3I_A{}^2R_F$

OR another simple solution by using the power flow P_{AG}=P_{in}-P_{SCL}-P_{core}

P_{AG} =$\sqrt{3}V_TI_L\cos\theta$ - P_{SCL}-P_{core}

= 23.67-0.4-0.567 kW

= 22.7 kW

(g) The power converted from electrical to mechanical form is

$$P_{\text{conv}} = (1-s)P_{\text{AG}} = 0.98*22.7 \text{ kW} = 22.25 \text{ kW}$$

(h) The synchronous speed of this motor is

$$n_{\text{sync}} = \frac{120f_{se}}{P} = \frac{120(60 \text{ Hz})}{4} = 1800 \text{ r/min}$$

$$\omega_{\text{sync}} = (1800 \text{ r/min})\left(\frac{2\pi \text{ rad}}{1 \text{ r}}\right)\left(\frac{1 \text{ min}}{60 \text{ s}}\right) = 188.5 \text{ rad/s}$$

Therefore the induced torque in the motor is

$$\tau_{\text{ind}} = \frac{P_{\text{AG}}}{\omega_{\text{sync}}} = \frac{22.7 \text{ kW}}{(1800 \text{ r/min})\left(\frac{2\pi \text{ rad}}{1 \text{ r}}\right)\left(\frac{1 \text{ min}}{60 \text{ s}}\right)} = 120.4 \text{ Nm}$$

(i) The output power of this motor is

$P_{\text{OUT}}=P_{\text{conv}}-P_{\text{mech}}-P_{\text{misc}}$ = 22.25 – 0.15 – 0.4 kW= 21.74 kW

The output speed is

$$n_m=(1-s)\ n_{\text{sync}}=(1-0.02)\ (1800\text{ r/min})=1764\text{ r/min}$$

Therefore the load torque is

$$\tau_{\text{load}}=\frac{P_{\text{OUT}}}{\omega_m}=\frac{21.74\text{ kW}}{(1764\text{ r/min})\left(\frac{2\pi\text{ rad}}{1\text{ r}}\right)\left(\frac{1\text{ min}}{60\text{ s}}\right)}=117.7\text{ N}\cdot\text{m}$$

(j) The overall efficiency is

$$\eta=\frac{P_{\text{OUT}}}{P_{\text{IN}}}\times 100\%=\frac{P_{\text{OUT}}}{3V_\phi I_A\cos\theta}\times 100\%$$

$$\eta=\frac{21.74\text{ kW}}{3(266\text{ V})(35.5\text{ A})\cos(33.2°)}\times 100\%=91.7\%$$

(k) The motor speed in revolutions per minute is 1764 r/min. The motor speed in radians per second is

$$\omega_m=(1764\text{ r/min})\left(\frac{2\pi\text{ rad}}{1\text{ r}}\right)\left(\frac{1\text{ min}}{60\text{ s}}\right)=184.7\text{ rad/s}$$